ME 323: FLUID MECHANICS-II

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Compressible Flow

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Properties at various Mach number

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

$$\rightarrow \frac{p_0}{p} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$$

$$\rightarrow \frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{1}{k-1}}$$

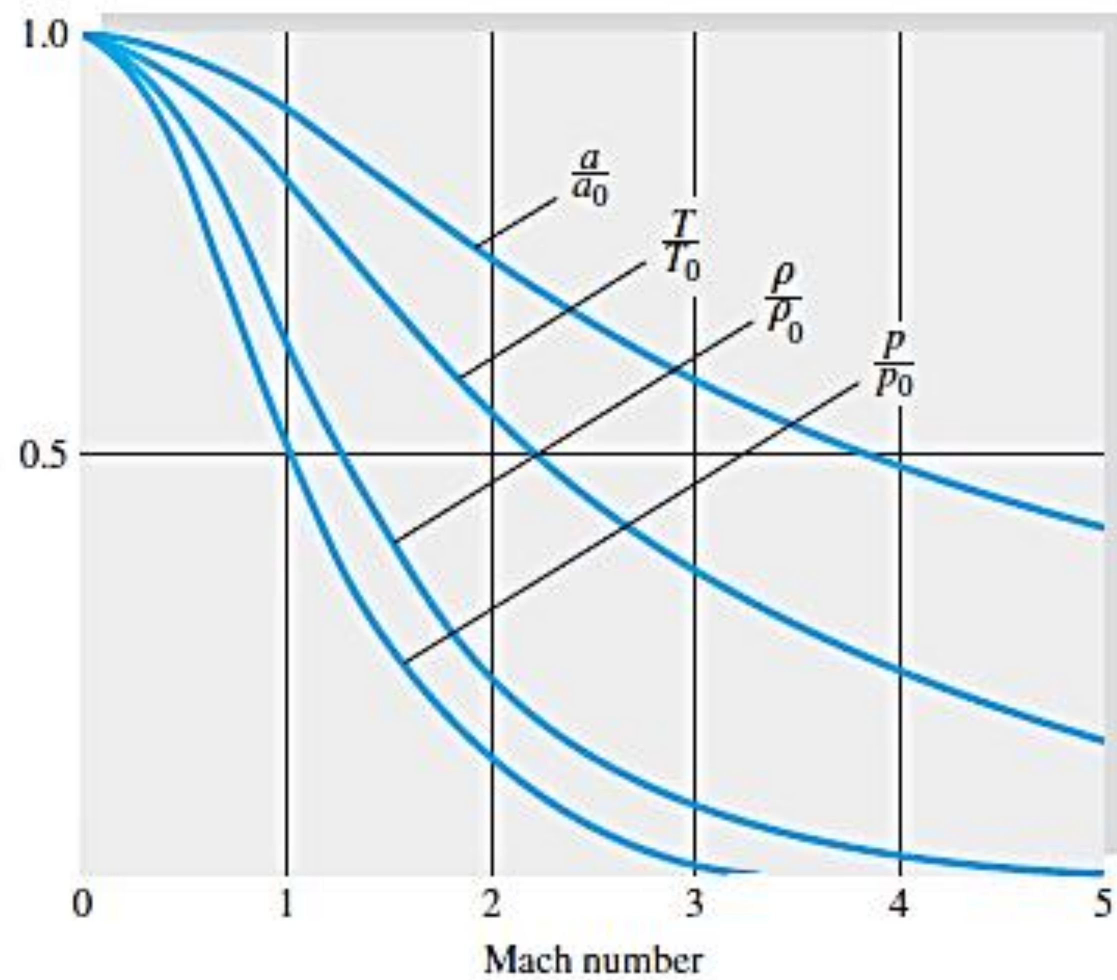


Fig. 9.3 Adiabatic (T/T_0 and a/a_0) and isentropic (p/p_0 and ρ/ρ_0) properties versus Mach number for $k = 1.4$.

are all gas flows compressible flows??

Not all gas flows are compressible flows, neither are all compressible flows gas flows.

Table: Variation of density with *M*

M	ρ_0/ρ	$\Delta\rho$
0.1	1.005	0.5%
0.2	1.02	2%
0.3	1.04	4%
0.4	1.08	8%
0.5	1.13	13%
1.0	1.58	58%
2.0	4.35	335%

$$\Rightarrow \frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{1}{k-1}}$$

For air k =1.4

General High speed/bullet train Shinkansen (Japan), TGV (France)

At low speeds, less than Mach number of about 0.3 (~100 m/s, 360 km/hr at STP), gas flows may be treated as incompressible flows since the density variations caused by the flow **are less than 5% which is insignificant in engineering sense.**

Isentropic flow with area changes

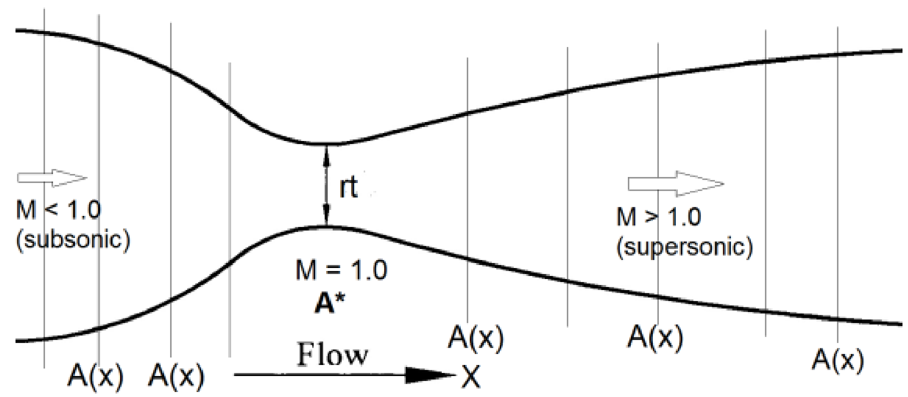
For compressible flow, due to change of area both the velocity and density are affected. For simplicity the following assumptions are made in deriving the **area-velocity relation for compressible flow**:

(i) One-dimensional flow i.e. $V = V(x)$ only

(ii) Steady state flow ($d/dt() = 0$)

(iii) Frictionless ideal flow ($\mu = 0$)

(iv) Adiabatic condition ($Q = W = 0$)



In case of steady 1-D flow, the mass continuity equation is:

$$\rho(x)V(x)A(x) = \dot{m} = \text{constant} = C$$

$$\Rightarrow \log \rho + \log V + \log A = \log C$$

$$\Rightarrow \frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \qquad (i)$$

(On differentiating)

Isentropic flow with area changes

Now the 1-D momentum equation **(Euler equation) in differential form**:

$$\frac{dp}{\rho} + VdV = 0 \qquad (ii)$$

And the equation for speed of sound in differential form:

$$a^2 = \frac{dp}{d\rho} \quad \rightarrow \quad dp = a^2 d\rho \qquad (iii)$$

Then equation (ii) comes as:

$$a^2 \frac{d\rho}{\rho} + VdV = 0 \quad \rightarrow \quad \frac{d\rho}{\rho} = -\frac{VdV}{a^2} \qquad (iv)$$

Use the above expression (iv) into equation (i)

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \qquad (i)$$

$$\Rightarrow -\frac{VdV}{a^2} + \frac{dV}{V} + \frac{dA}{A} = 0$$

$$\Rightarrow -\frac{V^2 dV}{a^2} + dV + \frac{VdA}{A} = 0 \qquad (v)$$

(multiply both side by V)

Isentropic flow with area changes

$$\Rightarrow -\frac{V^2 dV}{a^2} + dV + \frac{VdA}{A} = 0$$

$$\Rightarrow -M^2 dV + dV + \frac{V}{A} dA = 0 \qquad \because \text{Mach number}, M = \frac{V}{a}$$

$$\Rightarrow \frac{dV}{dA} = \frac{V}{A}\left[\frac{1}{(M^2 - 1)}\right]$$

This equation governs the shape of a nozzle or a diffuser in subsonic or supersonic isentropic flow.

This equation shows the relationship between change of area with change of velocity for different Mach number (***qualitative relation***).

There are two regimes for this relation:

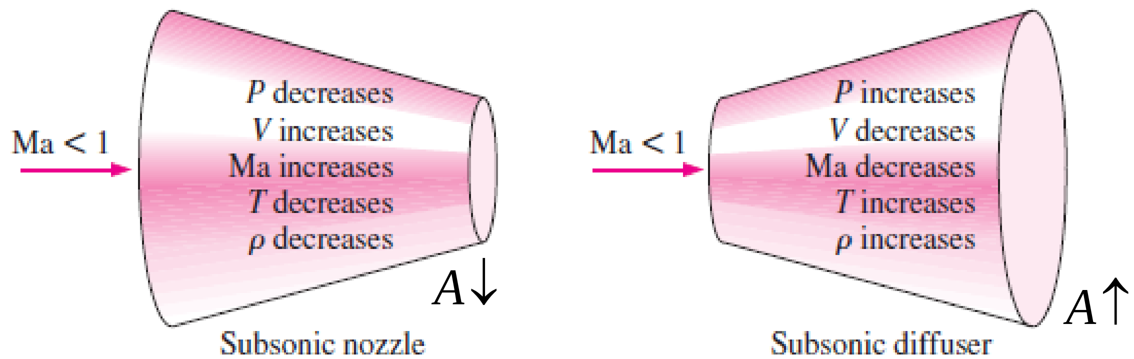
- $\otimes$ Subsonic flow, $M < 1.0 \quad \rightarrow \quad \frac{dV}{dA} = -ve$ ← **Area and velocity inversely proportional**
- $\otimes$ Supersonic flow, $M > 1.0 \quad \rightarrow \quad \frac{dV}{dA} = +ve$ ← **Area and velocity directly proportional**

Change of velocity with change of area

Isentropic flow with area changes

$$\Rightarrow \frac{dV}{dA} = \frac{V}{A}\left[\frac{1}{(M^2-1)}\right]$$

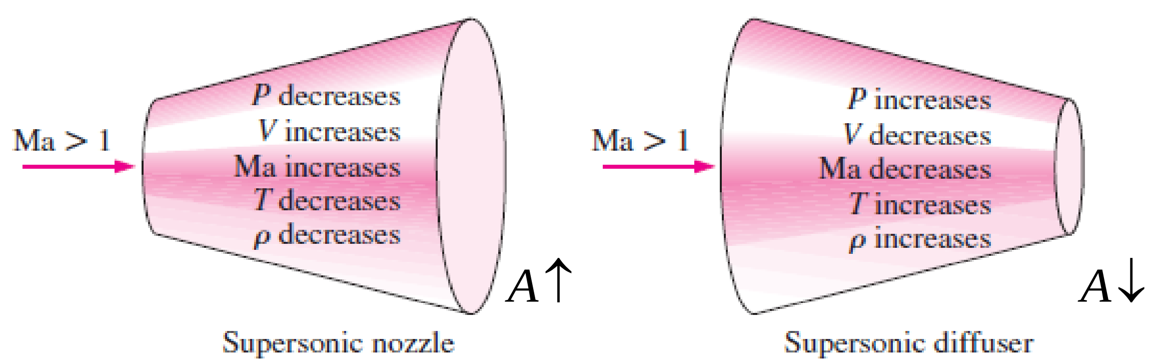
Subsonic flow



(*a*) Subsonic flow

$$\frac{dA}{dV} < 0 \ (-ve)$$

Supersonic flow



(*b*) Supersonic flow

$$\frac{dA}{dV} > 0 \ (+ve)$$

At **sonic point (*M* = 1.0)**,

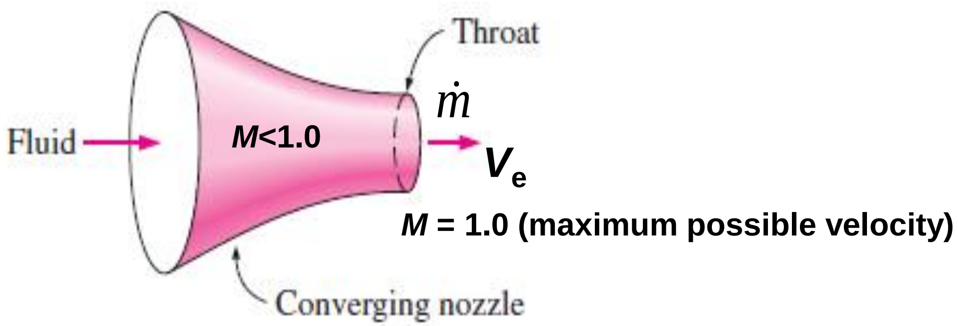
$$\Rightarrow \frac{dV}{dA} = \frac{V}{A}\left[\frac{1}{(M^2-1)}\right]$$

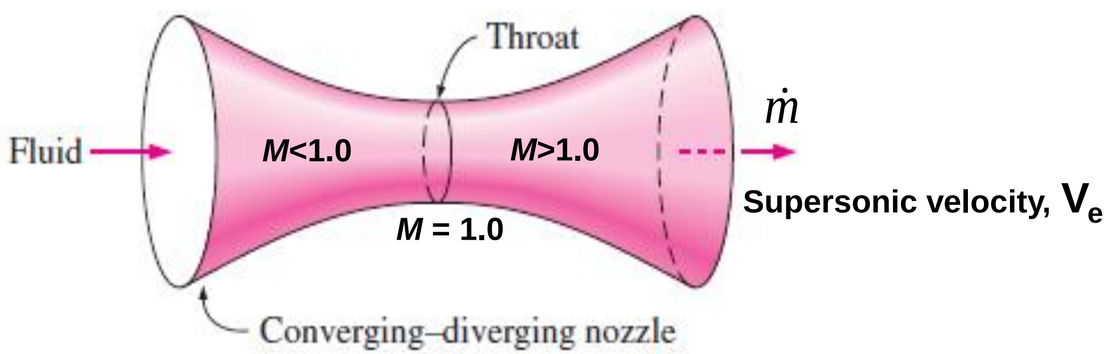
$$\rightarrow \quad \frac{\left(\dfrac{dV}{V}\right)}{\left(\dfrac{dA}{A}\right)} = \frac{1}{(M^2-1)}$$

$$\Rightarrow \frac{\left(\dfrac{dV}{V}\right)}{\left(\dfrac{dA}{A}\right)} = \infty \qquad ; \qquad \text{at sonic point } M = 1.0$$

Since infinite acceleration (dV/V) is physically impossible, the above mathematical formulation states that dV can be finite only when dA = 0 - that is a minimum area (**throat**).

The throat of a converging-diverging section can smoothly accelerate a subsonic (M <1.0) flow through sonic (M =1.0) to supersonic flow (M >1.0) as shown in figure:





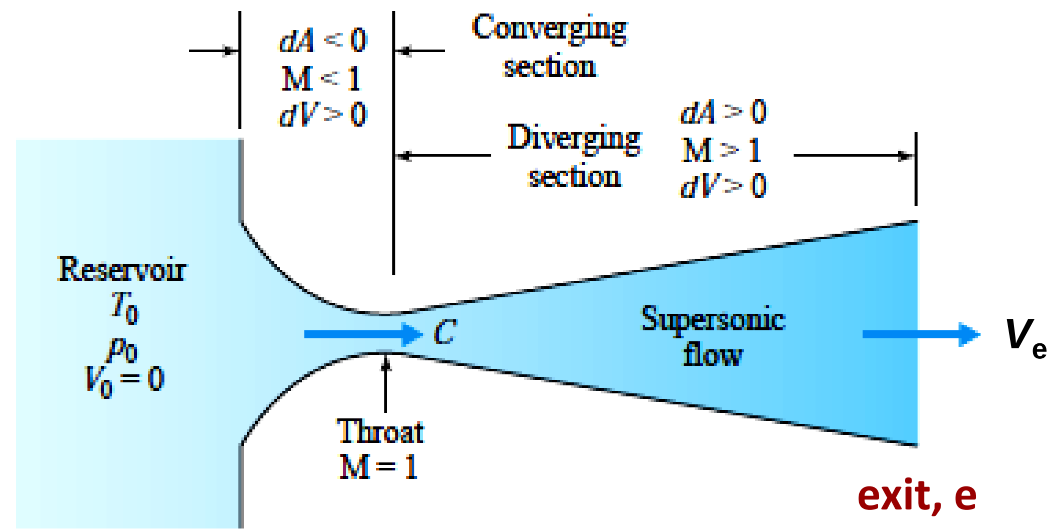


Fig. 9.4 Supersonic nozzle.

$$\text{Thrust, } T \approx \dot{m}V_e$$

$$\dot{m} = \rho_e A_e V_e$$

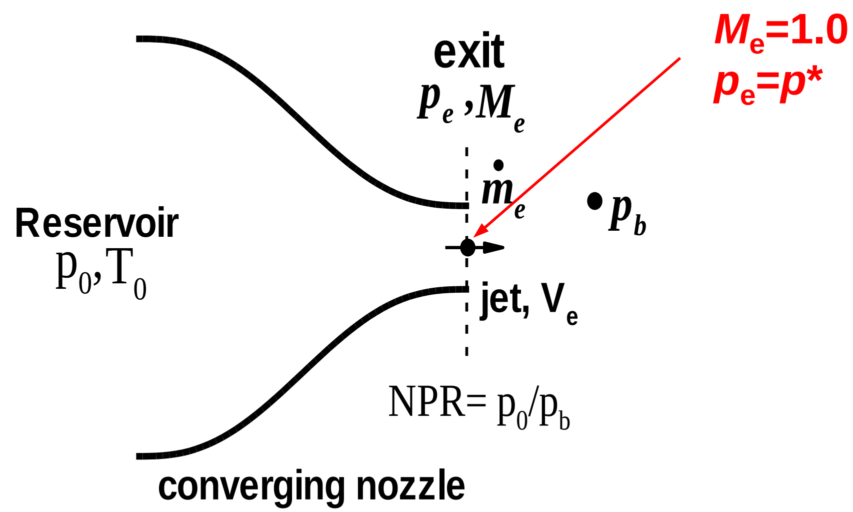
$$(T)_{\text{C–D nozzle}} > (T)_{\text{C nozzle}}$$

Critical values at sonic point (M=1.0)

The stagnation values (p_0, T_0, ρ_0) are useful reference conditions in a compressible flow, but of comparable usefulness are the conditions where the **flow is sonic, M = 1.0**.

The **sonic or critical properties** are denoted by asterisks: p^*, T^*, ρ^*. These are certain ratios of stagnation properties when M =1.0 (sonic point).

For air, k = 1.4;



$$\text{at } M = 1.0;\quad \frac{p_0}{p^*} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}} \rightarrow \frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \rightarrow \boxed{\frac{p^*}{p_0} = 0.5283}$$

$$\text{at } M = 1.0;\quad \frac{T_0}{T^*} = \left(1 + \frac{k-1}{2} M^2\right) \rightarrow \frac{T^*}{T_0} = \left(\frac{2}{k+1}\right) \rightarrow \boxed{\frac{T^*}{T_0} = 0.8333}$$

$$\text{at } M = 1.0;\quad \frac{\rho_0}{\rho^*} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{1}{k-1}} \rightarrow \frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \rightarrow \boxed{\frac{\rho^*}{\rho_0} = 0.6339}$$

Isentropic relations

Critical conditions

For air
k = 1.4